

Chemical Source Localization and Detection Using Interpolation method and the Complementary Spline Function Neural Network

Xiang Gao *, Levent Acar **

Dept. of Electrical and Computer Engineering, Missouri University of Science and Technology, Rolla, MO, USA

Abstract-Odor localization has been partially achieved in the previous papers by using knowledge of airflow, and a search on chemical sensing and reasoning. However the results are not specific, with the sensors can giving more data. If we arrange more sensors in the considered area or the sensors can mobile to gather more information at different locations, we need update the map of airflow and reasoning system to detect odor source. This paper presents a solution to the problem by introducing a second search stage using neural network. We proposed a new type neural network that uses hermite spline function being its activation function. In the experiment, the neural network can satisfy all of data from sensors and the weights are the parameters of the particle paths.

Key words: odor source detection, Spline interpolation, neural network algorithm

1. INTRODUCTION

The ability to locate of an odor/chemical source has many valuable applications. These applications include finding the source of dangerous substances such as airborne biological material, hazardous chemical, gas and other pollutants, in industrial and other settings; searching for survivors in earthquake-damaged buildings; detecting fire in its initial stages; locating unexploded mines and bombs.

These applications have been tackled by researchers in a variety of methods [1 and 2]. Most of their works have focused on mobile robots to detect plume and trace plume [3 and 4]. Odor dispersal occurs through carriage by the fluid current. Many researchers has used a chemical gradient method that is acquiring the plume firstly, and then moving upwind along the plume [5, 6, 7, and 8]. But, their limitation are that robots must follow the plume along its entire length, which is time consuming and may not be possible in some environment where a well-defined plume does not be formed[9 and 10]. In addition, the effects of objects and walls on both airflow and robot mobile are often neglected.

In this paper, we have attempted to overcome these limitations. By exploiting more information from the environment in addition to local wind and chemical reading, the airflow map and chemical distribution can be derived first, then basing the aforementioned result a 'sense-map-plan-act' control scheme is built [10 and 11].

However, when the sensor system collects more data, the problem of updated mapping is faced to be solved. Because the information is local, which does not affect the whole mapping; we just need update the airflow and chemical dispersion around the sensors that gather new data. Neural networks using splines function can update its weights to satisfy the new data from sensors.

2. PARTICLE PATHS MAPPING AND ODOR DISPERSAL

Using the sensors that can collect the sensor's position, wind velocity, chemical concentration, we can know the particle paths that describe the propagation in the environment. This map is the prerequisite for detection the odor source.

In this paper, we want to interpolate two nodes (points) (x_0, y_0) and (x_1, y_1) , and the two points are the locations of two sensors. Instead of using the interpolation $(x, y) \approx (x, H(x))$, we put x and y on more equal footing by using $(x, y) \approx (H_x(t), H_y(t))$ where $0 \leq t \leq 1$ is a parameter such

$$\text{that} \quad x_0 = H_x(0) \quad x_1 = H_x(1) \quad y_0 = H_y(0) \quad y_1 = H_y(1) \quad 2.1$$

In other words, we construct two separate Hermite polynomials each approximating x and y . Eq. 2.1 provide us

with x_0, x_1, y_0, y_1 , however we also need $\frac{\partial x_0}{\partial t}, \frac{\partial x_1}{\partial t}, \frac{\partial y_0}{\partial t}, \frac{\partial y_1}{\partial t}$. These are indirectly provided in form of guide

points (x_0^+, y_0^+) and (x_1^-, y_1^-) . The derivatives are taken as the slope of the lines drawn between the nodes and the guide points. Namely

$$\left. \frac{\partial y}{\partial x} \right|_{t=0} = \frac{y_0^+ - y_0}{x_0^+ - x_0} \quad \left. \frac{\partial y}{\partial x} \right|_{t=1} = \frac{y_1 - y_1^-}{x_1 - x_1^-} \quad 2.2$$

Note that the + and – superscripts dictate which side of the difference the points lie on. The problem is that these are the derivatives of y with respect to x, but we need the derivatives of x and y with respect to t. these however aren't too hard to find using the identity

$$\frac{\partial y}{\partial x} = \frac{\frac{\partial y}{\partial t}}{\frac{\partial x}{\partial t}} \quad 2.3$$

This suggests that we take

$$\begin{aligned} \left. \frac{\partial x}{\partial t} \right|_{t=0} &= \frac{\partial x_0}{\partial t} = x_0^+ - x_0 & \left. \frac{\partial y}{\partial t} \right|_{t=0} &= \frac{\partial y_0}{\partial t} = y_0^+ - y_0 \\ \left. \frac{\partial x}{\partial t} \right|_{t=1} &= \frac{\partial x_1}{\partial t} = x_1 - x_1^- & \left. \frac{\partial y}{\partial t} \right|_{t=1} &= \frac{\partial y_1}{\partial t} = y_1 - y_1^- \end{aligned} \quad 2.4$$

Now we can proceed to construct the two Hermite polynomials in the usual way.

$$\begin{aligned} H_x(t) &= ([1 - 2(t-0)L'_{1,0}(0)]L_{1,0}(t)^2)x_0 + ((t-0)L_{1,0}(t)^2)(x_0^+ - x_0) + ([1 - 2(t-1)L'_{1,1}(1)]L_{1,1}(t)^2)x_1 \\ &+ ((t-1)L_{1,1}(t)^2)(x_1 - x_1^-) \\ &= ((x_0 + x_0^+) - (x_1^- + x_1))t^3 - ((x_0 + 2x_0^+) - (x_1^- + 2x_1))t^2 - (x_0 - x_0^+)t + x_0 \end{aligned} \quad 2.5$$

And completely analogously for $H_y(t)$

$$H_y(t) = ((y_0 + y_0^+) - (y_1^- + y_1))t^3 - ((y_0 + 2y_0^+) - (y_1^- + 2y_1))t^2 - (y_0 - y_0^+)t + y_0 \quad 2.6$$

Once we obtain the sensory information, we start with an approximation of the particle path. We configure paths that go through the sensor locations, such that the paths satisfy the locations as well as the differentials. This approach leads to a parametric cubic-polynomial representation of the path in terms of a variable t. We use the cubic Hermite spline with the end point differentials weighted three times, such that

$$\begin{aligned} x(t) &= (2(x(0) - x(1)) + 3(\delta x(0) + \delta x(1)))t^3 + 3(x(1) - x(0)) - 3(\delta x(1) + 2\delta x(0))t^2 + 3\delta x(0)t + x(0), \\ y(t) &= (2(y(0) - y(1)) + 3(\delta y(0) + \delta y(1)))t^3 + 3(y(1) - y(0)) - 3(\delta y(1) + 2\delta y(0))t^2 + 3\delta y(0)t + y(0). \end{aligned}$$

Where the parametric curve starts at one sensor location at $x(0)$, $y(0)$ and ends at the other sensor location at $x(1)$, $y(1)$ as t goes from 0 to 1.

We compute the expected concentration value along the computed path and compare it with the actual sensed concentration value. Based on the error and the measured gradient concentration, we determine a new location perpendicular to the initial path where the expected and sensed concentration values match.

We then compute the corrected path going through one of the sensors and the new location. When we repeat the process forwards from one sensor and backwards from another one, we end up getting two consistent paths with correct concentration values.

In the next step of the extrapolation, we fill the whole room with secondary paths. For the secondary paths that are between two adjacent primary paths, we determine the normal (perpendicular lines to the tangents of the paths), and use the intersection points of the normal to generate a secondary path. We assign the average values of the particle concentrations and the concentration gradients on these paths. For the secondary paths that are on the outside regions of the primary paths, we use similar normal extensions, but we extrapolate the particle concentrations and the concentration gradients. Figure 1 shows the path extensions as well as the whole room coverage with primary and the secondary paths.

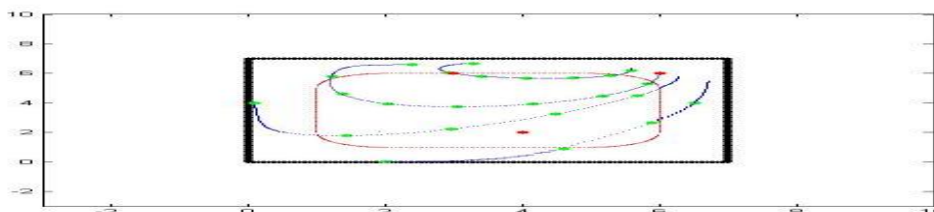


Fig. 2.1 Primary and secondary air-borne particle paths going through two sensors.

3. NEURAL NETWORK BASED ON SPLINE BASIS FUNCTION

In this paper, the theory of spline approximation is adequately combined with the neural network principle by these advantages of spline function. The neural network model based on m times spline basis function is constructed, where w_j is the weight of neural network and the $\phi_j(t)$ is spline basis function. That is $\phi_j(t) = t^j, j = 0, 1, \dots, m$, the hidden layer neuron incentive function $t \in [0, 1]$. Suppose the weight matrix $W = [w_0, w_1, \dots, w_m]^T$ and the incentive matrix is $\Phi(t) = [\phi_0, \phi_1, \dots, \phi_m]^T$. So the output of the neural network is $X(x, y) = \sum_{j=0}^m w_j \phi_j(t) = W^T \Phi(t)$. Comparing with the cubic Hermite splines, the weights of the neural networks are the parameters of the splines function. The error function is $e(k) = X'(t_n) - f'(t_n), n = 0, 1, \dots, N+2$, where $n = N+2$, N is the number of new sensor points, $f'(t_n)$ is from the new data and $X'(t_n)$ is from the spline function. Therefore, the error function is the difference between splines interpolation and the real data.

The performance index is $J = \frac{1}{2} \sum_{k=0}^n e^2(k)$. and the weight adjustment is $W(k+1) = W(k) + \eta J(k) \Phi(k)$.

3.1 Convergence Theorem of Neural Network Model

Let η be the learning rate. Then the neural network algorithm is convergent, when $0 \leq \eta \leq \frac{2}{\sum_{j=0}^m |\phi_j(t)|^2}$, where m is the number of hidden layer neurons.

Proof:

Let the Lyapunov function

$$V(k) = \frac{1}{2} e^2(k) \quad \text{Then } \Delta V(k) = \frac{1}{2} e^2(k+1) - \frac{1}{2} e^2(k),$$

And

$$e(k+1) = e(k) + \Delta e(k) = e(k) + \left(\frac{\partial e(k)}{\partial W} \right)^T \frac{\partial e(k)}{\partial W},$$

$$\Delta W = -\eta e(k) \frac{\partial e(k)}{\partial W},$$

So,

$$\Delta e(k) = -\eta e(k) \left(\frac{\partial e(k)}{\partial W} \right)^T \frac{\partial e(k)}{\partial W} = -\eta e(k) \left\| \frac{\partial e(k)}{\partial W} \right\|_2^2,$$

Where $\|\cdot\|_2 = \sqrt{\sum |\cdot|^2}$ is the Euclidean norm. So Lyapunov function can be written as

$$\begin{aligned} \Delta V(k) &= \frac{1}{2} [e(k) + \Delta e(k)]^2 - \frac{1}{2} e^2(k) \\ &= \Delta e(k) [e(k) + \frac{1}{2} \Delta e(k)] \\ &= -\eta e(k) \left\| \frac{\partial e(k)}{\partial W} \right\|_2^2 [e(k) - \frac{1}{2} \eta e(k) \left\| \frac{\partial e(k)}{\partial W} \right\|_2^2] \\ &= \left\| \frac{\partial e(k)}{\partial W} \right\|_2^2 e(k) \left(-\eta + \frac{1}{2} \eta^2 \left\| \frac{\partial e(k)}{\partial W} \right\|_2^2 \right) \end{aligned}$$

To make the neural network algorithm convergent, we have

$$-\eta + \frac{1}{2} \eta^2 \left\| \frac{\partial e(k)}{\partial W} \right\|_2^2 \leq 0, \eta > 0,$$

That is $0 < \eta \leq \frac{2}{\left\| \frac{\partial e(k)}{\partial W} \right\|_2^2}$.

We get,

$$\frac{\partial e(k)}{\partial W} = \left(\frac{\partial e(k)}{\partial X(t)} \right) \left(\frac{\partial X(t)}{\partial W} \right) = -\Phi(t).$$

According to $\Phi(t) = [\phi_0, \phi_1, \dots, \phi_m]^T$, it can be proved

$$\left\| \frac{\partial e(k)}{\partial W} \right\|_2^2 = \|\Phi(t)\|_2^2 = \sum_{j=0}^m |\phi_j|^2.$$

when $0 \leq \eta \leq \frac{2}{\sum_{j=0}^m |\phi_j(t)|^2}$, we have $\Delta V < 0$.

Consequently, it is shown that the neural network algorithm is convergent.

3.2 Algorithm of Neural Network Model

According to the neural network model based on the spline function which is discussed above, we get the following neural network algorithm for update the airflow path.

Take the learning rate satisfying theorem, to make sure convergence of the neural network;

Calculate the output $f'(t_n)$ of neural network:

$$X(x, y) = \sum_{j=0}^m w_j \phi_j(t) = W^T \Phi(t);$$

Calculate the error function:

$$e(k) = X'(t_n) - f'(t_n), n=0,1,\dots,N+2,$$

Where, $X'(t_{n,k})$ demonstrate the gradient of the particle path, and $f'(t_n)$ is given by the real data;

Calculate the performance index of the neural network:

$$J = \frac{1}{2} \sum_{k=0}^n e^2(k);$$

Adjust the neural network weight: $W(k+1) = W(k) + \eta J(k) \Phi(k)$.

As a test case, we assume to use n sensors to collect information in the environment. In figure 6, however, when we make one more sensor in the search are, the aforementioned method of mapping can only be changed around the new sensor. The core of this proposed neural network method is updating the parameters of cubic hermit splines using all the data.

In the figure 1, the top two sensors are primary sensor. Using the data from these sensors we can get the airflow path and chemical dispersion that are shown in the figure. When the new data from the bottom sensor, we can consider neural network method that can get the updated map by training the weights of the network. In this case, the t is considered to the input and the position (x, y) is considered to the output, and the error term is the difference between spline functions that are derived from output and the real data from sensors. In the fig. 3.1, the new map using the new data from the bottom sensor is shown below.

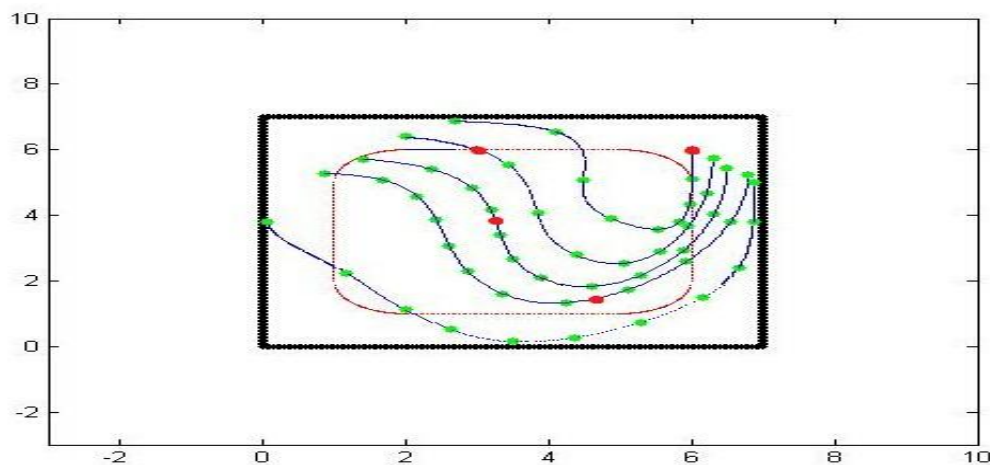


Fig. 3.1 The new particle path map using 3 sensors

4. NUMERICAL EXAMPLES

In this case, we assume there are 4 sensors in the room. Depending on the rules from section two, we can judge the multi-link is 1-2-3-4. It means the wind go through sensor 1 first, then sensor 2 and 3, last sensor 4. Through the map, we can get the particle paths and know the position, velocity, concentration of every point on the paths. The fig. 4.1 shows the concentration distribution along the particle path. The circles denote the sensors, and the y axis shows the value of chemical concentration. The odor source is located between sensor 1 and sensor 2. In down flow, the chemical concentration is decayed smoothly with a small rate, but in the upstream, the chemical concentration is decayed dramatically, because the wind blows most of chemical particle downstream

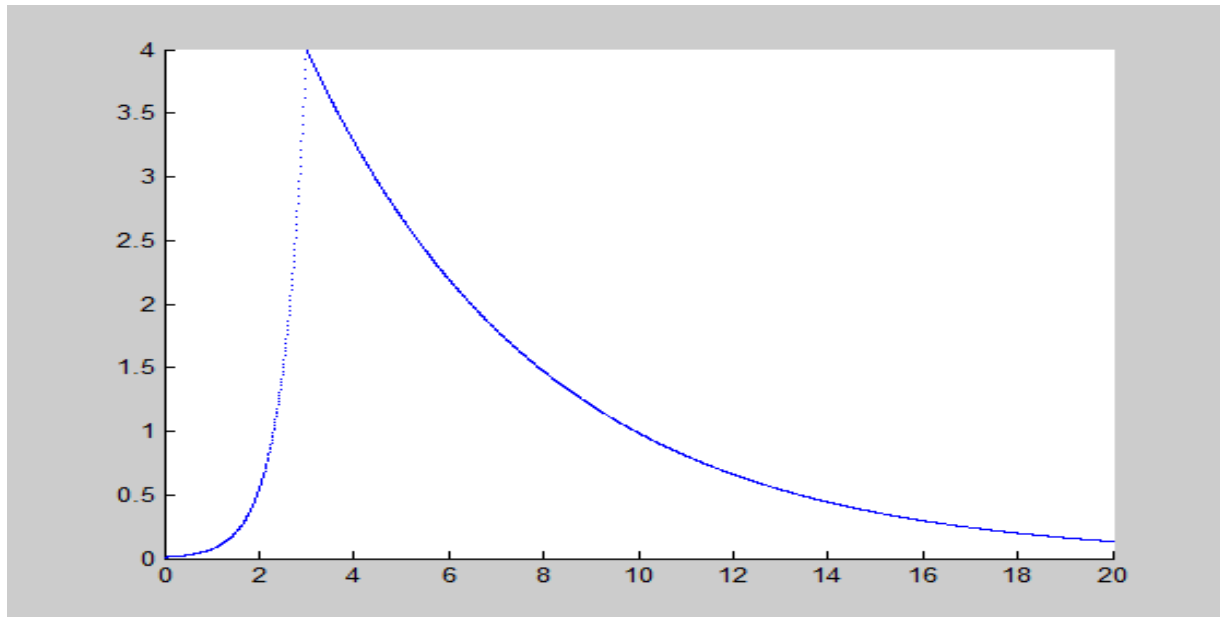


Fig. 4.1 The chemical concentration on the particle path

Case 1: ($S_n > S_0$ or $S_n < S_0$) after cancelling one sensor, by the other sensors we get the new particle map and new chemical dispersal. If the new chemical concentration (S_n) on this cancelled sensor point is higher (or lower) than the actual value (S_0) the sensor get, this sensor is upstream (or downstream) of the odor source. It's called critical sensor.

Case 2: ($S_n \approx S_0$) After cancelling one sensor, by the other sensors we get the new particle map and new chemical dispersal. If the new chemical concentration (S_n) on this cancelled sensor point is close to the actual value (S_0) the sensor get, this sensor is far from the odor source, this sensor is not the critical sensor.

When the sensor 1 is cancelled, the calculated chemical concentration at the location of sensor 1 becomes higher than the original. The result can be seen in figure 8. By the theorem 1, sensor 1 is up to the odor source and is the upper critical sensor. We apply same method to sensor 2, 3 and 4. The chemical concentration on the point of sensor 2 becomes higher than the original. The result can be seen in fig. 4.2. By the theorem 1, sensor 2 is down the odor source and is the lower critical sensor. The chemical concentration on the point of sensor 3 and 4 are equal with the original. By the theorem 2, sensor 3 and 4 are not close to the source and are not critical sensors. Through the above analysis, the odor source should be located between sensor 1 and sensor 2.

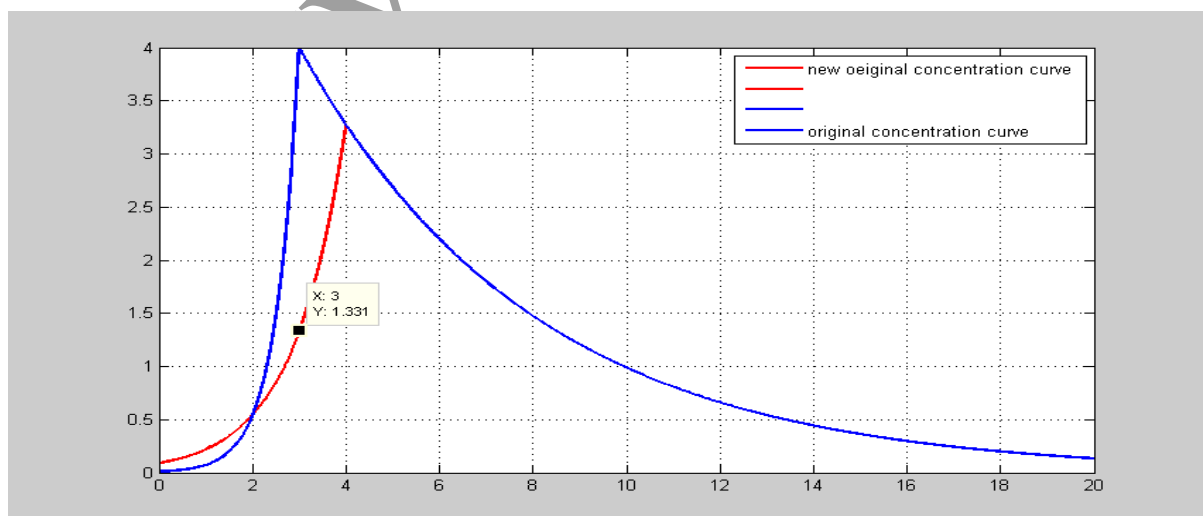


Fig. 4.2 Concentration Curves Using All Sensors and Using 3 Sensors

Through the above process, the searched region is restricted between two critical sensors. In the fig. 4.3, we can see the shadowed area is our considered area between sensor 1 and 2. However, we know the odor source is a point source, so we hope the source can be located in a smaller region. Next, the reduction of the region is needed to learn.

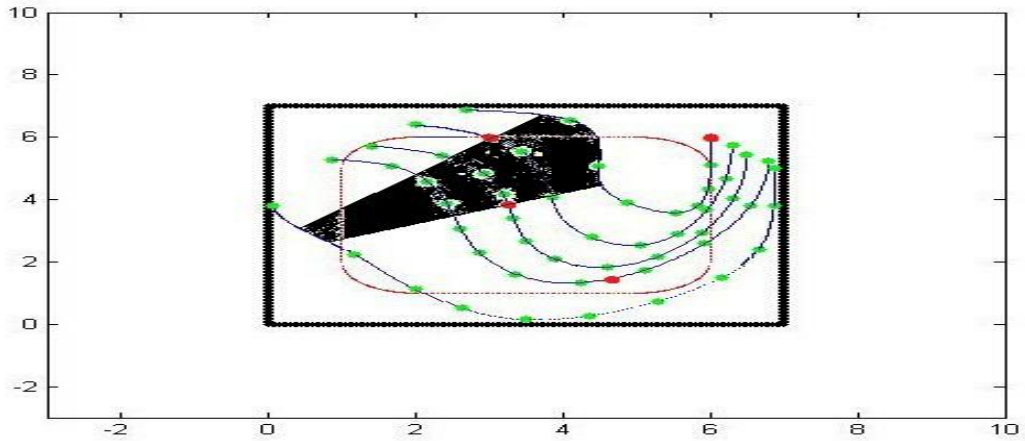


Fig. 4.3 The region selected by critical sensors

We determine the normal (perpendicular lines to the tangents of the paths) on the critical sensor nodes. Then the normal lines generate intersection points with the parallel paths. We hope we can know the odor source exists between which two particle paths.

We do A consumption: comparing the chemical concentrations on the every intersections, the two paths that contain the two highest concentration on the intersection points are the critical paths. The region surrounded by the critical paths and two perpendicular lines is the most-likely hood for odor source.

The concentration on the intersections can be calculated by the odor propagation along the particle path. We know the concentration on the sensor node and know the decay rate along the paths. In this paper, we think it obeys the exponential decay. The reason why we select the down critical sensor is because the chemical concentration from the up critical sensor is always a small number.

In the test case, by calculating the concentrations on every intersection, we get the critical paths. In the fig. 4.4, the shadowed region is most-likely region that contains the odor source.

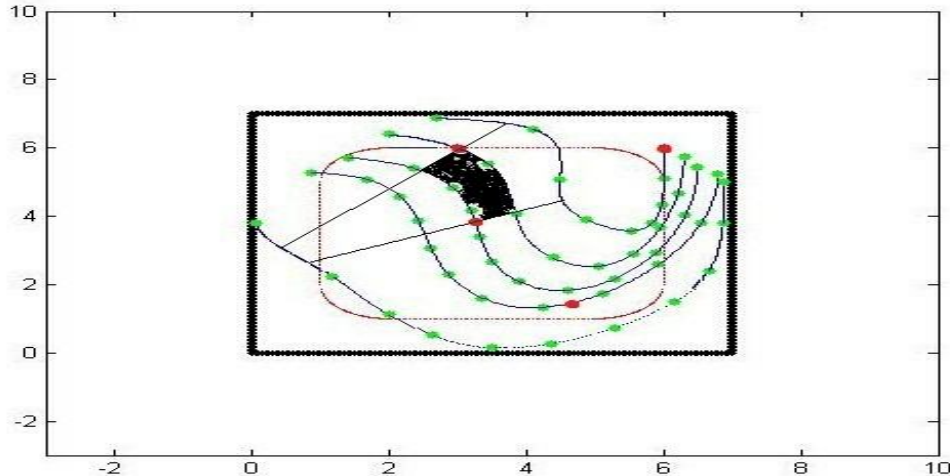


Fig. 4.4 The Most-Likely Region Selected by Critical Sensors

CONCLUSION

There are many useful and humanitarian applications that can locate the source of a chemical source. Currently, the majority of work in this area uses reactive control schemes that track an odor plume along its entire length, which is slow and difficult in cluttered environments. This paper employs a high-level control scheme. The interpolation and extrapolation method is used to model the particle path in the sensors' environment. Then a reasoning system use the path model to get the velocity, chemical concentration at any point on the map and predict the most probable locations of the odor source. This approach has been shown to be effective for odor localization in a known environment, without the need for the robot to travel to the source.

With the further development there is great potential for this approach to lead to many valuable applications by generalization to a wider range of environmental configurations. The paper present development to solve the problem there exist obstacles and opening in the environment. The approach gives the mode of the particle path

surrounding the obstacles and openings. The development has successfully applied in general environment, because the propagation of the chemical particle can go through obstacles and opening in actual case. In addition, this paper is the first example of using interpolation and extrapolation method to model the particle path that applied in a real environment.

REFERENCES

- [1] G. Eason, B. Noble, and I. N. Sneddon, "On certain integrals of Lipschitz-Hankel type involving products of Bessel functions," *Phil. Trans. Roy. Soc. London*, vol. A247, pp. 529–551, April 1955.
- [2] G. O. Young, "Synthetic structure of industrial plastics (Book style with paper title and editor)," in *Plastics*, 2nd ed. vol. 3, J. Peters, Ed. New York: McGraw-Hill, 1964, pp. 15–64.
- [3] W.-K. Chen, *Linear Networks and Systems* (Book style). Belmont, CA: Wadsworth, 1993, pp. 123–135.
- [4] H. Poor, *An Introduction to Signal Detection and Estimation*. New York: Springer-Verlag, 1985, ch. 4.
- [5] E. H. Miller, "A note on reflector arrays (Periodical style—Accepted for publication)," *IEEE Trans. Antennas Propagat.*, to be published.
- [6] Y. Yorozu, M. Hirano, K. Oka, and Y. Tagawa, "Electron spectroscopy studies on magneto-optical media and plastic substrate interfaces(Translation Journals style)," *IEEE Transl. J. Magn.Jpn.*, vol. 2, Aug. 1987, pp. 740–741 [Dig. 9th Annu. Conf. Magnetics Japan, 1982, p. 301.
- [7] M. Young, *The Technical Writers Handbook*. Mill Valley, CA: University Science, 1989.
- [8] J. U. Duncombe, "Infrared navigation—Part I: An assessment of feasibility (Periodical style)," *IEEE Trans. Electron Devices*, vol. ED-11, pp. 34–39, Jan. 1959.
- [9] S. Chen, B. Mulgrew, and P. M. Grant, "A clustering technique for digital communications channel equalization using radial basis function networks," *IEEE Trans. Neural Networks*, vol. 4, pp. 570–578, July 1993.
- [10] X. Gao, L. Acar, Detection and Tracking of Odor source in Sensor Networks Using Reasoning System, *Journal of Automation and Control Engineering (JOACE, ISSN:2301-3702)*.
- [11] X. Gao, L. Acar, Multi-sensor Integration to map odor distribution for detecting chemical source, *Sensors*, July, 2016.